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ENERGY MANAGEMENT POLICY SELECTION IN SMART GRIDS: A CRITIC-CoCoSo METHOD WITH Lq* q-rung ORTHOPAIR MULTI-FUZZY SOFT SETS

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Abstract:
In response to the energy crisis and the global push for sustainability, modern power grids are increasingly integrating renewable energy, plug-in electric vehicles, and energy storage systems. This evolution demands an advanced energy management system capable of handling the variability of renewable resources, uncertainties in electric vehicle performance, fluctuating electricity prices, and dynamic load conditions. To address these challenges, our study introduces a novel decision-making framework that leverages a new score function for comparing q-rung orthopair multi-fuzzy soft numbers. This approach employs the Criteria Importance Through Inter-criteria Correlation (CRITIC) method to determine objective weights while simultaneously incorporating subjective preferences through an integrated weighting scheme. The framework is further enhanced by applying the Combined Compromise Solution (CoCoSo) method within the Lq* q-rung orthopair multi-fuzzy soft decision-making structure to select optimal energy management policies. Extensive sensitivity analysis confirms the robustness and effectiveness of the proposed methodology, offering a promising solution for efficient energy management in modern power systems.

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1. INTRODUCTION

Energy management systems (EMS) optimize energy use in smart grids. EMS plays a crucial role in efficiency, with studies examining administrative strategies and optimization techniques [1]. For renewable integration, robust optimization has been applied to plug-in-vehicle-storage-grid systems, and a multi-criteria framework has been proposed for selecting storage technologies. Optimization in smart microgrids has been explored through cost-emission models, and EMS limitations in resilient microgrids have been studied [2]. Fuzzy-based methodologies and optimization techniques further enhance EMS decision-making in smart grids.

Multi-criteria decision-making (MCDM) frequently involves multiple options and evaluation criteria in everyday scenarios. MCDM challenges can be categorized into individual and group decision-making problems, where defining precise numerical values for attributes is often problematic. To address these complexities, Zadeh's fuzzy logic [3] is designed to handle imprecise and uncertain data and enhance decision-making in Energy Management Systems (EMS), which involve complex energy sources, dynamic demand, and operational uncertainties. Modeling EMS as a fuzzy Multi-Criteria Decision-Making (MCDM) problem [4,5] allows for better optimization and adaptability, addressing the limitations of

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traditional MCDM methods in managing conflicting criteria.

This innovation addressed vagueness and uncertainty, leading to numerous extensions, including intuitionistic, interval-valued, hesitant, neutrosophic, and Pythagorean fuzzy sets [6,7]. These advancements have broad applications in science, technology, economics, medical science, and engineering, solidifying their importance in solving complex problems. Among the novel developments in fuzzy set theory, multi-fuzzy sets (MFSs) [8,9] have gained attention for addressing challenges like pixel color and image recognition, which are difficult to resolve using other extensions. In 2017, Yager [10,11] introduced the concept of q-rung orthopair fuzzy sets (q-ROFS), a generalization of Pythagorean fuzzy sets (PFS) [12]. When q=1, the concept simplifies to IFS, and when q=2, it corresponds to PFS. Extensive research [13] has since examined IFS, PFS, MFS, and q-ROFS, expanding their theoretical and practical applications.

Soft sets, first proposed by Molodtsov [14], offered a flexible decision-making framework by incorporating parameter values. Maji et al. [13,15] extended this concept by defining fuzzy soft sets (FSS) and intuitionistic fuzzy soft sets (IFSS), which Peng et al. [16] further generalized into Pythagorean fuzzy soft sets (PFSS). Yang et al. [17] integrated multi-fuzzy sets with soft set models to develop multi-fuzzy soft sets (MFSS), which have since been expanded into various extensions [18-20]. Das and Kar [21] introduced intuitionistic multi-fuzzy soft sets (IMFSS) for solving complex decision-making problems.

One significant application area for these theories is ranking or ordering linguistic expressions such as "good," "decent," and "tremendous." For instance, educators use qualitative terms to rank student performance, which helps clarify relative positions. Lattice theory, introduced by Birkhoff [22] in the 1930s, has significantly contributed to fields like computer science, engineering, communication systems, and mathematics. Concepts such as ideals, morphisms, modular lattices, and distributive lattices have been extensively explored and enriched by scholars like George Gratzer, who examined their fundamentals and applications. Addressing this need, lattice-ordered (LO) soft sets [23] were introduced in 2015, providing a framework to analyze linguistic terms with defined rankings. Over time, this concept evolved into intuitionistic fuzzy soft sets [24], anti-lattice ordered double-framed soft sets [25], with the latter emerging in 2018.

Weight determination methods are classified as subjective or objective. Subjective weights rely on decision-makers' expertise, influencing alternative ranking, while objective weights use mathematical models and assessment data, ensuring consistency. The CRITIC method, a key objective approach, quantifies criteria through standard deviations and inter-criteria relevance, proving effective across domains. The CoCoSo method, introduced by Yazdani et al. [26], enhances decision-making by integrating Simple Additive Weighting (SAW) and Exponentially Weighted Product (EWP) models through comparability sequences. Widely applied in technology selection, human resources, and supplier evaluation, CoCoSo is effective but faces challenges with complex datasets.

An intuitionistic fuzzy soft CoCoSo-CRITIC framework [27] has been introduced for CCN cache placement, enhancing decision-making under uncertainty. Expanding on this, Pythagorean [28] and hesitant [29] fuzzy soft approaches have been applied for fog computing cache replacement and IoE company evaluation, respectively, addressing hesitation in decision-making and optimizing storage efficiency. A picture fuzzy soft CRITIC-CoCoSo model [30] has also been introduced for supplier selection in Industry 4.0, handling uncertainty and conflicting criteria effectively.

Vimala et al. [31] recently advanced the IMFSS model with q-ROMFSS framework. Pethaperumal et. al. [32] combines the strengths of q-ROMFS sets with the advantages of lattice ordering of parameters. Building on this foundation, this article integrates the $Lq*$ q-rung orthopair multi-fuzzy soft set with CRITIC and CoCoSo methods for selecting energy management policies in smart grid systems. These enhancements improve the flexibility and precision of decision-making, enabling more accurate modeling and analysis in complex scenarios characterized by data uncertainty and vagueness. Fig. 1 illustrates the hierarchical structure of the proposed Lq* q-ROMFSS, outlining its key components and decision-making framework.

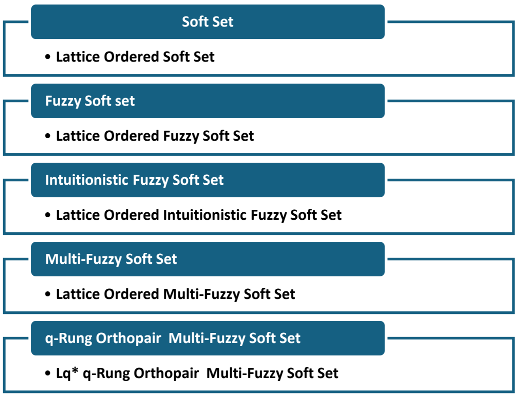


Fig. 1. Hierarchical Structure of Proposed Lq* q-ROMFSS

The research gaps are in the existing CRITIC-CoCoSo MCDM approaches, which fail to effectively capture multi-dimensional fuzzy soft information, limiting their ability to produce reliable outcomes. This study introduces a new perspective to address key challenges which are comparing q-rung orthopair multi-fuzzy soft numbers using a score function; identifying the most beneficial energy management policy under uncertain conditions and optimizing the energy management policy selection process through MCDM strategies.

A new CRITIC-CoCoSo methodology is formulated within the Lq* q-ROMFSS framework to efficiently tackle MCDM challenges. The approach follows a three-phase process:

1. In the first phase, a refined score function is developed to process q-rung orthopair multi-fuzzy soft data by integrating multi-membership, non-membership, and hesitation degrees, ensuring a more accurate decision evaluation.
2. The second phase involves determining objective weights using the Criteria Importance Through Inter-criteria Correlation (CRITIC) method. Simultaneously, an integrated weighting scheme is established by balancing subjective weight preferences with objective weight calculations. To address the issue of low discrimination, a CoCoSo algorithm is implemented within the Lq* q-ROMFSS framework, enhancing the effectiveness of the decision-making process.
3. The proposed approach is applied to evaluate and select an energy management policy in a smart grid system, addressing literature gaps and offering a practical solution. Comparative and sensitivity analyses confirm its reliability and effectiveness.

The study is structured as follows:

- **Section 2** reviews the foundational concepts of q-ROFS, q-ROMFS, and q-ROMFSS.
- **Section 3** introduces a novel score function for q-ROMFSN, addressing multi-membership, non-membership, and hesitation and proposes an algorithm for Lq∗ q-ROMFS-based CoCoSo MCDM method.
- **Section 4** presents a practical application for selecting energy management policies in smart grids.
- **Section 5** validates the proposed methodology through sensitivity and comparative analysis.
- **Section 6** concludes with key insights and the broader implications of the proposed methodologies.

2. PRELIMINARIES

This section explores the key concepts of Lq* q-rung orthopair multi-fuzzy soft set.

Definition: 2.1 A fuzzy set Q_F on V is defined by mapping $\varsigma: V \to [0,1]$, where $\varsigma(r)$ for every $r \in V$, represents the degree of that object to which that element is related to the fuzzy set. It can be expressed as $Q_F=\{(r, \varsigma(r)): r \in V\}$.

Definition: 2.2 Let V be a universal set, and G be a set of attributes. A soft set S over V is defined as a mapping: $S: H \to P^V$ where P^V denotes the collection of all subsets of V.

Definition: 2.3 A multi-fuzzy set (M^kF-set) M_F of dimension k (a positive integer) over V is defined as $M_F = \left\{\left(r, \zeta^p{}_{M_F}(r)\right): r \in V\right\}$ where $\zeta^p{}_{M_F}: V \to [0,1], p = 1,2,\dots,k$ is the multi-MemF of M_F, and the set of all MFS of dimension k is denoted as $M^kFS(V)$.

Definition:2. 4 A q-ROFS R_F over V is defined as $R_F = \left\{\left(r, \zeta_{R_F}(r), \partial_{R_F}(r)\right): r \in V\right\}$ for each $r \in V$ the functions $\zeta_{R_F}: V \to [0,1], \partial_{R_F}: V \to [0,1]$ denotes the MemF and NMemF of R_F respectively with the constraint that $(\zeta_{R_F})^q + (\partial_{R_F})^q \leq 1$ with $q \geq 1$. For each $r \in V$, the indeterminacy degree is given by:

$$\Pi_{T_F}(r) = \sqrt[q]{1 - (\zeta_{T_F}(r))^q - (\partial_{T_F}(r))^q}.$$

Definition: 2.5 A q-rung orthopair multi-fuzzy set (q-ROMFS) T_F of dimension k over V is defined as follows:

$$T_F = \left\{\left(r, (\zeta^1{}_{T_F}(r), \partial^1{}_{T_F}(r)), \quad (\zeta^2{}_{T_F}(r), \partial^2{}_{T_F}(r)), \dots, (\zeta^k{}_{T_F}(r), \partial^k{}_{T_F}(r))\right), r \in V\right\}$$

for each $r \in V$ the functions $\zeta^p{}_{T_F}: V \to [0,1], \partial^p{}_{T_F}: V \to [0,1]$, $p = 1,2,\dots,k$ denotes the multi- MemF and multi-NMemF of T_F respectively with the constraint that $(\zeta^p{}_{T_F})^q +$

$(\partial^p{}_{T_F})^{\ q} \leq 1$. For each $r \in V$, the indeterminacy degree is given by:

$\Pi^p_{S_{T_F}}(r) = \sqrt[q]{1 - (\zeta^p{}_{T_F}(r))^{\ q} - (\partial^p{}_{T_F}(r))^{\ q}}$.

The collection of all q-ROM^{k}FS set of dimension k over V is indicated as q-ROM^{k}FS$^{(V)}$.

Definition: 2.6 A pair (S_{T_F}, G) is a q-ROMFS set of dimension k over V if $S_{T_F}: G \to \mathrm{q-ROM^kFS^{(V)}}$ and it is defined as follows:

$$(S_{T_F}, G) = \left\{ \left(h, S_{T_F}(h)\right), h \in H \sqsubseteq G, \ S_{T_F}(h) \, \epsilon \, \mathrm{q-ROM^kFS^{(V)}} \right\}$$

where is:

$$S_{T_F}(h) = \left\{ \left(r, \left(\zeta^p{}_{S_{T_F(h)}}(r), \partial^p{}_{S_{T_F(h)}}(r) \right) \right), r \in V, p = 1,2,\ldots,k \ and \ q \geq 1 \right\}.$$

For simplicity, $S_{T_F}(r) = \left(\zeta^p{}_{S_{T_F(h)}}(r), \partial^p{}_{S_{T_F(h)}}(r) \right)$ is named as a q-rung orthopair multi-fuzzy soft number.

Definition: 2.7 A q-rung orthopair multi-fzzy soft set (q-ROM^{k}FSS) T_F of dimension k defined over V is known as Lq* (Lattice Ordered) q-ROMFS set if for each $h_1, h_2 \in H \sqsubseteq G$ such that $h_1 \leq h_2$ implies $S_{T_F}(h_1) \sqsubseteq S_{T_F}(h_2)$.

i.e., $\left(\zeta^p{}_{S_{T_F(h_1)}}(r) \leq \zeta^p{}_{S_{T_F(h_2)}}(r), \ \partial^p{}_{S_{T_F(h_1)}}(r) \geq \partial^p{}_{S_{T_F(h_2)}}(r) \right)$, for all $r \in V$ and p =1,2,.. k. Then the matrix representation of Lq* q-ROMFS set is represented as follows:

$$\aleph = [\aleph_{ij}] = \begin{pmatrix} (\zeta^1{}_{S_{T_F(h_1)}}(r_1), \ \partial^1{}_{S_{T_F(h_1)}}(r_1)) & (\zeta^1{}_{S_{T_F(h_2)}}(r_1), \ \partial^1{}_{S_{T_F(h_2)}}(r_1)) & \cdots & (\zeta^1{}_{S_{T_F(h_y)}}(r_1), \ \partial^1{}_{S_{T_F(h_y)}}(r_1)) \\ (\zeta^1{}_{S_{T_F(h_1)}}(r_2), \ \partial^1{}_{S_{T_F(h_1)}}(r_2)) & (\zeta^1{}_{S_{T_F(h_2)}}(r_2), \ \partial^1{}_{S_{T_F(h_2)}}(r_2)) & \ldots & (\zeta^1{}_{S_{T_F(h_y)}}(r_2), \ \partial^1{}_{S_{T_F(h_y)}}(r_2)) \\ \vdots & \ldots & \ddots & \vdots \\ (\zeta^1{}_{S_{T_F(h_1)}}(r_x), \ \partial^1{}_{S_{T_F(h_1)}}(r_x)) & (\zeta^1{}_{S_{T_F(h_2)}}(r_x), \ \partial^1{}_{S_{T_F(h_2)}}(r_x)) & \cdots & (\zeta^1{}_{S_{T_F(h_y)}}(r_1), \ \partial^1{}_{S_{T_F(h_y)}}(r_x)) \end{pmatrix}_{x*y}$$

3. Lq* q-rung ORTHOPAIR FUZZY SOFT-MCDM USING CRITIC-CoCoSo METHOD

3.1 Novel Score Function of q-ROMFSS

This section presents an advanced score function tailored for q-ROM^{k}FS numbers. The refined function considers the levels of multi-membership (multi-memF), non-membership (non-memF), and hesitation.

Definition: 3.1 A score function of q-ROMFSNs $S_{T_F}(r) = \left(\zeta^p{}_{S_{T_F(h)}}(r), \partial^p{}_{S_{T_F(h)}}(r) \right)$ can be defined as:

$$\mathfrak{H}(S_{T_F(h_j)}(r_i)) = \frac{\sum_{p=1}^{k} \left\{ (\zeta^p{}_{S_{T_F(h_j)}}(r_i))^q - (\partial^p{}_{S_{T_F(h_j)}}(r_i))^q + \left(\frac{e^{(\zeta^p{}_{S_{T_F(h_j)}}(r_i))^q - (\partial^p{}_{S_{T_F(h_j)}}(r_i))^q}}{e^{(\zeta^p{}_{S_{T_F(h_j)}}(r_i))^q - (\partial^p{}_{S_{T_F(h_j)}}(r_i))^q} + 1} - \frac{1}{2} \right) \left(\Pi^p_{S_{T_F(h_j)}}(r_i) \right)^q \right\}}{k}$$

where $q \geq 1$ and $S_{T_F(h_j)}(r_i) \in [-1,1]$.

Definition: 3.2 For any two q-ROMFSNs $S_{T_F(h_1)}(r_1)) = \left(\zeta^p{}_{S_{T_F(h_1)}}(r_1), \partial^p{}_{S_{T_F(h_1)}}(r_1) \right)$ *and* $S_{T_F(h_2)}(r_1)) = \left(\zeta^p{}_{S_{T_F(h_2)}}(r_1), \partial^p{}_{S_{T_F(h_2)}}(r_1) \right)$ of dimension k, then

1. If $\mathfrak{H}(S_{T_F(h_1)}(r_1))) < \mathfrak{H}(S_{T_F(h_2)}(r_1)))$, then $S_{T_F(h_1)}(r_1)) < S_{T_F(h_2)}(r_1))$.
2. If $\mathfrak{H}(S_{T_F(h_1)}(r_1))) > \mathfrak{H}(S_{T_F(h_2)}(r_1)))$, then $S_{T_F(h_1)}(r_1)) > S_{T_F(h_2)}(r_1))$.
3. If $\mathfrak{H}(S_{T_F(h_1)}(r_1))) = \mathfrak{H}(S_{T_F(h_2)}(r_1)))$, then
   a. If $\Pi(S_{T_F(h_1)}(r_1))) < \Pi(S_{T_F(h_2)}(r_1)))$, then $S_{T_F(h_1)}(r_1)) < S_{T_F(h_2)}(r_1))$.
   b. If $\Pi(S_{T_F(h_1)}(r_1))) > \Pi(S_{T_F(h_2)}(r_1)))$, then $S_{T_F(h_1)}(r_1)) > S_{T_F(h_2)}(r_1))$.

3.2 Lq* q-Rung Fuzzy Soft MCDM via CRITIC-CoCoSo

In Multi-Criteria Decision-Making (MCDM) problems, criteria are fundamental, with their associated weights reflecting the information embedded within each criterion, referred to as "objective weights". The CRITIC method provides a robust way to calculate these weights by evaluating the importance of each criterion through standard deviation and measuring the conflict between criteria using correlation coefficients. This method is extended to the Lq* q-ROMFSS framework, enabling the handling of evaluation data represented as q-ROMFSNs. In this context, $S_{T_F(h_j)}(r_i)$ denotes the q-ROMFSNs of the i^{th}

alternative under the j^{th} criteria, while w_j^0 represents the objective weight for the j^{th} criteria. The process of determining these weights is guided by both beneficial and non-beneficial criteria (BC and N-BC).

Let $V = \{r_1, r_2, \dots, r_x\}$ represent the set of alternatives, and $\mathrm{H} = \{\mathrm{h}_1, \mathrm{h}_2, \dots, \mathrm{h}_\mathrm{y}\}$ denote the set of parameters with dimension k . Additionally, let $\varpi = \{\varpi_1, \varpi_2 \dots, \varpi_y\}$, satisfying $\sum_{j=1}^{y} \varpi_j = 1, 0 \leq \varpi_j \leq 1$. Suppose the evaluation values of the i^{th} alternative r_i concerning the j^{th} criteria h_j are expressed as the q-ROMFSN $S_{T_F}(r) = \left(\zeta^p{}_{S_{T_F(h)}}(r), \partial^p{}_{S_{T_F(h)}}(r)\right)$, as illustrated in Table 1.

Table 1. Representation of Lq* q-rung orthopair multi-fuzzy soft matrix

	h_1	h_2	$\cdots$	h_y
P_1	$(\zeta^1{}_{S_{T_F(h_1)}}(r_1), \partial^1{}_{S_{T_F(h_1)}}(r_1))$	$(\zeta^1{}_{S_{T_F(h_2)}}(r_1), \partial^1{}_{S_{T_F(h_2)}}(r_1))$	$\cdots$	$(\zeta^1{}_{S_{T_F(h_y)}}(r_1), \partial^1{}_{S_{T_F(h_y)}}(r_1))$
P_2	$(\zeta^1{}_{S_{T_F(h_1)}}(r_2), \partial^1{}_{S_{T_F(h_1)}}(r_2))$	$(\zeta^1{}_{S_{T_F(h_2)}}(r_2), \partial^1{}_{S_{T_F(h_2)}}(r_2))$	$\cdots$	$(\zeta^1{}_{S_{T_F(h_y)}}(r_2), \partial^1{}_{S_{T_F(h_y)}}(r_2))$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
P_x	$(\zeta^1{}_{S_{T_F(h_1)}}(r_x), \partial^1{}_{S_{T_F(h_1)}}(r_x))$	$(\zeta^1{}_{S_{T_F(h_2)}}(r_x), \partial^1{}_{S_{T_F(h_2)}}(r_x))$	$\cdots$	$(\zeta^1{}_{S_{T_F(h_y)}}(r_x), \partial^1{}_{S_{T_F(h_y)}}(r_x))$

3.2.1 *CRITIC-Algorithm*

Step 1: Calculate the score function $\aleph = [\aleph_{ij}]_{x*y}$ of each q-ROM^{k}FSN $S_{T_{F(h)}}(r) = \left(\zeta^p{}_{S_{T_F(h)}}(r), \partial^p{}_{S_{T_F(h)}}(r)\right)$.

$$\aleph_{ij} = \frac{\sum_{p=1}^{k}\left\{(\zeta^p{}_{S_{T_F(h_j)}}(r_i))^q - (\partial^p{}_{S_{T_F(h_j)}}(r_i))^q + \left(\frac{e^{(\zeta^p{}_{S_{T_F(h_j)}}(r_i))^q - (\partial^p{}_{S_{T_F(h_j)}}(r_i))^q}}{e^{(\zeta^p{}_{S_{T_F(h_j)}}(r_i))^q - (\partial^p{}_{S_{T_F(h_j)}}(r_i))^q}+1} - \frac{1}{2}\right)\left(\Pi^p_{S_{T_F(h_j)}}(r_i)\right)^q\right\}}{k} \tag{1}$$

Step 2: Standardize the score matrix $\aleph$ to obtain its normalized form as $\aleph' = [\aleph'_{ij}]_{x*y}$:

$$\aleph'_{ij} = \begin{cases} \frac{\aleph_{ij} - \aleph_j^-}{\aleph_j^+ - \aleph_j^-}, if\ j \in BC \\ \frac{\aleph_j^+ - \aleph_{ij}}{\aleph_j^+ - \aleph_j^-}, if\ j \in N - BC \end{cases} \tag{2}$$

where $\aleph_j^- = \min \aleph_{ij}$ and $\aleph_j^+ = \max \aleph_{ij}$.

Step 3: Calculate the standard deviation for each criterion to assess its variability.

$$\sigma_j = \sqrt{\left(\frac{\sum_{i=1}(\aleph'_{ij} - \overline{\aleph}_j)^2}{x}\right)} \text{ where } \overline{\aleph}_j = \frac{\sum_{i=1}^{x} \aleph'_{ij}}{x} \tag{3}$$

Step 4: Determine the correlation coefficients for each pair of criteria to evaluate their relationships.

$$\delta_{jl} = \frac{\sum_{i=1}^{x}(\aleph'_{ij} - \overline{\aleph}_j)((\aleph'_{il} - \overline{\aleph}_l)}{\sqrt{\sum_{i=1}^{x}\left(\aleph'_{ij} - \overline{\aleph}_j\right)^2 \sum_{i=1}^{x}(\aleph'_{il} - \overline{\aleph}_l)^2}} \tag{4}$$

Step 5: Calculate the information value for each criterion to assess its impact on the decision-making process.

$$z_j = \sigma_j \sum_{l=1}^{y}(1 - \delta_{jl}) \tag{5}$$

Step 6: Calculate the j^{th} objective weight to determine the corresponding criterion's relative importance.

$$\varpi_j = \frac{z_j}{\sum_{j=1}^{y} z_j} \tag{6}$$

Step 7: Extract Integrated Weights through a Non-Linear Weighted Aggregation Method.

Surjective weights, which are directly assigned by decision-makers (DMs), are denoted as $w = \{w_1, w_2 \dots, w_y\}$, satisfying $\sum_{j=1}^{y} w_j = 1, 0 \leq w \leq 1$. Objective weights, which are indirectly computed by using Eq. (5), are represented as $\varpi = \{\varpi_1, \varpi_2 \dots, \varpi_y\}$, adhering to $\sum_{j=1}^{y} \varpi_j = 1,_j = 1, 0 \leq \varpi_j \leq 1$.

The integrated weights $\xi = \{\xi_1, \xi_2, \dots, \xi_y\}$ are then derived as:

$$\xi_j = \frac{\varpi_j * w_j}{\sum_{j=1}^{y} \varpi_j * w_j} \tag{7}$$

3.2.2 *CoCoSo-Method*

The Combined Compromise Solution (CoCoSo), developed by Yazdani et al. [26], is an advanced method used in Multi-Criteria Decision-Making (MCDM). It is derived from integrating the Simple Additive Weighting (SAW) methods and Exponentially Weighted Product (EWP), representing a compromise solution framework. In particular, the Lq* q-ROMFSS-CRITIC-CoCoSo method can be described in Fig. 2.

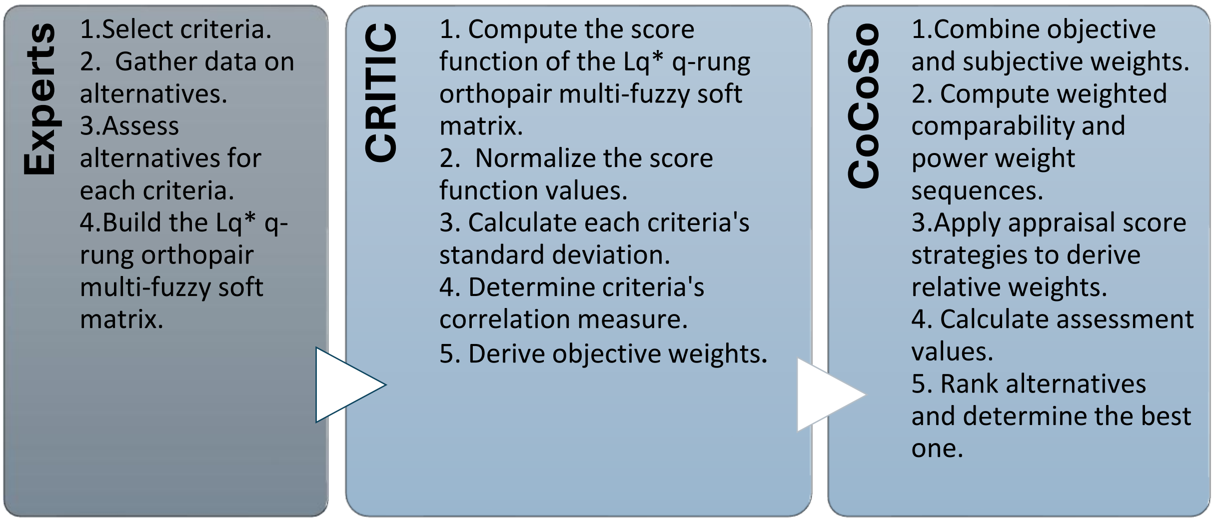


Fig. 2. Flowchart for the Proposed Lq* q-ROMFSS-CRITIC-CoCoSo Methodology

3.2.3 *Algorithm: Lq* q-ROMFSS CRITIC-CoCoSo*

Step 1: Generate the matrix $\beth = [\beth_{ij}]_{x*y}$ using the linguistic terms presented in Table 3.

Step 2: Transform the linguistic matrix into Lq* q-ROMFSS (S_{T_F}, G) , as outlined in Table 1.

Step 3: Calculate the score function $F = [F_{ij}]_{x*y}$ of each q-ROM^{k}FS$_f$N $S_{T_{F(h)}}(r) = \left(\zeta^p{}_{S_{T_F(h)}}(r), \partial^p{}_{S_{T_F(h)}}(r)\right)$ by Eq.(1).

Step 4: Standardize the score matrix U to obtain its normalized form as $\aleph' = [\aleph'_{ij}]_{x*y}$ by Eq.(2).

Step 5: Calculate the standard deviations of each criteria using Eq.(3).

Step 6: Determine the correlation measure for each pair of criteria to evaluate using Eq.(4)

Step 7: Calculate the informational value for each criterion using Eq.(5).

Step 8: Calculate the jth objective weight using Eq.(6).

Step 9: Calculate the integrated weight ξ by Eq.(7).

Step 10: Calculate the sum of the weighted comparability sequence as A_i:

$$A_i = \sum_{j=1}^{y} \xi_j * \aleph'_{ij} \tag{8}$$

Step 11: Calculate the sum of power weight of comparability sequence as B_i

$$B_i = \sum_{j=1}^{y} {\aleph'_{ij}}^{\xi_j} \tag{9}$$

Step 12: The three aggregation strategies formulated by Eqs. (10)-(12):

$$\gamma_{ia} = \frac{A_i + B_i}{\sum_{i=1}^{x} A_i + B_i} \tag{10}$$

$$\gamma_{ib} = \frac{A_i}{\min_i A_i} + \frac{B_i}{\min_i B_i} \tag{11}$$

$$\gamma_{ic} = \frac{\lambda A_i + (1-\lambda) B_i}{\lambda \max_i A_i + (1-\lambda) \max_i B_i}, 0 \le \lambda \le 1 \tag{12}$$

Where γ_{ia} indicates the average of WSM and WPM, γ_{ib} indicates the sum of relative scores of WPM and WSM about the optimal, and γ_{ic} reflects the scores representing a balanced compromise between WPM and WSM.

Step 13: Obtain the evaluation value γ_i by Eq.(14):

$$\gamma_i = (\gamma_{ia}\gamma_{ib}\gamma_{ic})^{\frac{1}{3}} + \frac{1}{3}(\gamma_{ia} + \gamma_{ib} + \gamma_{ic}) \tag{13}$$

Step 14: Order the alternatives in descending order according to the assessment value γ_i.

4. CASE STUDY: EVALUATING ENERGY MANAGEMENT POLICIES FOR SMART GRID OPTIMIZATION

Smart grids integrate advanced technologies to optimize the generation, distribution, and consumption of energy. They play a vital role in incorporating renewable energy sources such as solar, wind, and hydroelectric power, contributing to sustainability. However, the intermittent nature of these renewable sources introduces grid instability and fluctuations in energy supply, making it essential to implement optimal energy management policies.

These policies, including demand response programs, energy storage systems, and real-time data analytics, enhance grid performance by:

- Balancing supply and demand to prevent instability.
- Reducing energy wastage through smart scheduling.
- Facilitating the effective integration of renewable energy sources.
- Enhancing grid resilience to avoid outages.

Given the growing complexity of smart grids, evaluating the effectiveness of different energy management policies is crucial. This study applies the *Lq** q-ROMFSS method to rank and select optimal energy management strategies based on key performance parameters. The diagrammatic representation of alternatives and criteria is shown in Fig. 3.

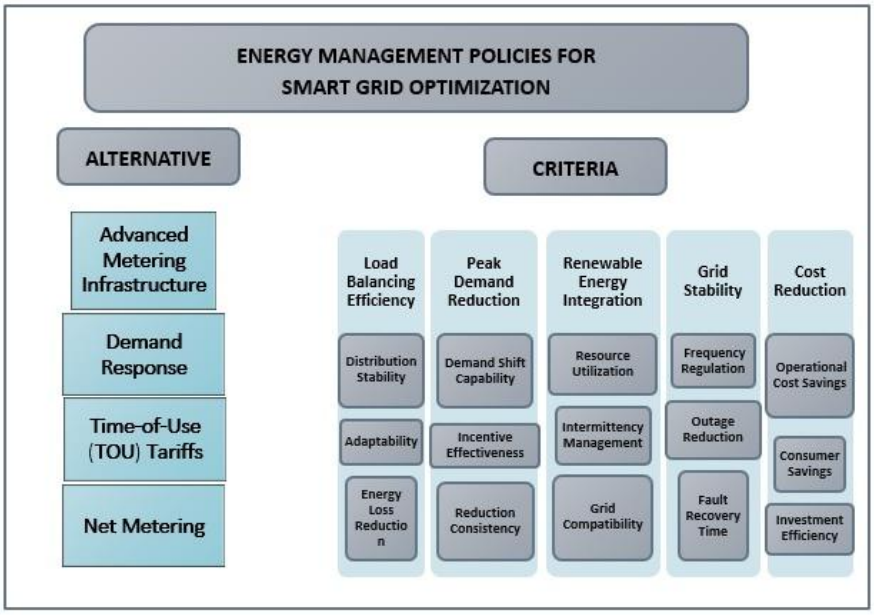


Fig. 3. Information of Alternatives and Criteria

4.1 Example

A mid-sized urban area with a peak electricity demand of 500 MW faces challenges related to grid congestion, high energy costs, and renewable energy intermittency. The local utility company aims to implement a set of energy management policies to:

- Optimize load balancing and system efficiency.
- Encourage consumer participation in energy conservation.
- Improve real-time monitoring and demand forecasting.

To address these challenges, four energy management policies are evaluated:

1. **P1 –** Advanced Metering Infrastructure (AMI) – Highest-rated for real-time monitoring and demand forecasting.
2. **P2 –** Demand Response (DR) – Effective in reducing peak demand and improving cost efficiency.
3. **P3 –** Time-of-Use (TOU) Tariffs – Encourages consumer participation but depends on user adaptability.
4. **P4 –** Net Metering – Promotes renewable integration but requires regulatory support.

4.1.1 Policy Evaluation Criteria

The effectiveness of these policies is assessed using a criteria set (H) determined by domain experts, prioritizing system efficiency, reliability, and sustainability before cost optimization. The criteria are as follows:

1. Load Balancing Efficiency (h_1) – Ensures matching of supply with demand, minimizing wastage.
2. Peak Demand Reduction (h_2) – Reduces grid stress and operational costs during peak periods.
3. Renewable Energy Integration (h_3) – Supports sustainability by efficiently incorporating clean energy sources.
4. Grid Stability (h_4) – Ensures system reliability and enhances overall performance.
5. Cost Reduction (h_5) – Secondary to technical and environmental goals but essential for long-term savings.

These criteria set $H=\{h_1,h_2,h_3,h_4,h_5\}$ determined by domain experts are ordered hierarchically to prioritize system efficiency, reliability, and sustainability before cost optimization, using the Lq∗ q-ROMFSS information. The relative importance of these criteria is represented by the weights W = $(0.10, 0.15, 0.20, 0.25, 0.30)^T$, reflecting the prioritization of the criteria in the evaluation process. The criteria are ordered as $h_1 \leq h_2 \leq h_3 \leq h_4 \leq h_5$ and its multi-dimensional evaluation is expressed in Table 2.

Table 2. The multi-dimensional assessment of energy management policy parameters

Criteria	Multi-dimensional Description
Load Balancing Efficiency (h_1)	**Distribution Stability** – The extent to which energy is distributed uniformly across nodes during high-demand periods.
	Adaptability – How quickly the policy adapts to sudden changes in demand patterns.
	Energy Loss Reduction – Measured by the reduction in transmission and distribution losses.
Peak Demand Reduction (h_2)	**Demand Shift Capability** – The policy's ability to shift energy usage from peak to off-peak hours.
	Incentive Effectiveness – How well the policy motivates users to reduce consumption during peak periods.
	Reduction Consistency – The degree to which peak demand is consistently reduced across multiple cycles.
Renewable Energy Integration (h_3)	**Resource Utilization** – The percentage of renewable energy used versus available renewable capacity.
	Intermittency Management – The ability to handle fluctuations in renewable energy sources like solar or wind.
	Grid Compatibility – How well the renewable energy sources are integrated into existing grid infrastructure without disruptions.
Grid Stability (h_4)	**Frequency Regulation** – The policy's impact on maintaining a stable frequency (e.g., 50 Hz or 60 Hz).
	Outage Reduction – The ability to prevent power outages during stress conditions.
	Fault Recovery Time – The speed at which the system recovers from faults or disruptions caused by sudden demand or supply shifts.
Cost Reduction (h_5)	**Operational Cost Savings** – Reduction in expenses like energy storage, distribution, and maintenance.
	Consumer Savings – The financial impact of the policy on end-users' energy bills.
	Investment Efficiency – The cost savings ratio to the initial investment required for policy implementation.

Table 3. The comparison evaluation table of the linguistic term and q-ROMFSN

Linguistic Term	Extremely Low	Very Low	Low	Medium	Above Medium	Medium High	High	Very High
Abbreviation	EL	VL	L	M	AM	MH	H	VH
q-ROMFSN	(0,1)	(0.2,0.9)	(0.5,0.6)	(0.6,0.5)	(0.7,0.4)	(0.8,0.3)	(0.9,0.2)	(1,0)

In the subsequent discussion, the suggested algorithm (λ=0.5) will be used to identify the best energy management policy by utilizing Lq∗ q-ROMFS set information.

Step 1: Generate the matrix $\beth = [\beth_{ij}]_{x*y}$ using the linguistic terms in Table 3.

$$\begin{bmatrix} (M,M,AM) & (M,M,MH) & (M,MH,MH) & (AM,MH,H) & (MH,H,H) \\ (L,M,M) & (M,AM,AM) & (M,AM,MH) & (AM,MH,H) & (MH,H,VH) \\ (M,AM,AM) & (M,AM,MH) & (M,AM,H) & (MH,H,VH) & (MH,H,VH) \\ (L,M,M) & (M,AM,AM) & (MH,H,H) & (MH,H,VH) & (H,H,VH) \end{bmatrix} \tag{14}$$

Step 2: Transform the linguistic matrix into an Lq∗ q-ROMFS set information based on Table 1.

$$\begin{bmatrix} <(0.6,0.5),(0.6,0.5),(0.7,0.4)> & <(0.6,0.5),(0.6,0.5),(0.8,0.3)> & <(0.6,0.5),(0.8,0.3),(0.8,0.3)> & <(0.7,0.4),(0.8,0.3),(0.9,0.2)> & <(0.8,0.3),(0.9,0.2),(0.9,0.2)> \\ <(0.5,0.6),(0.6,0.5),(0.6,0.5)> & <(0.6,0.5),(0.7,0.4),(0.7,0.4)> & <(0.6,0.5),(0.7,0.4),(0.8,0.3)> & <(0.7,0.4),(0.8,0.3),(0.9,0.2)> & <(0.8,0.3),(0.9,0.2),(1,0)> \\ <(0.6,0.5),(0.7,0.4),(0.7,0.4)> & <(0.6,0.5),(0.7,0.4),(0.8,0.3)> & <(0.6,0.5),(0.7,0.4),(0.9,0.2)> & <(0.8,0.3),(0.9,0.2),(1,0)> & <(0.8,0.3),(0.9,0.2),(1,0)> \\ <(0.5,0.6)(0.6,0.5),(0.6,0.5)> & <(0.6,0.5),(0.7,0.4),(0.7,0.4)> & <(0.8,0.3),(0.9,0.2),(0.9,0.2)> & <(0.8,0.3),(0.9,0.2),(1,0)> & <(0.9,0.2),(0.9,0.2),(1,0)> \end{bmatrix} \tag{15}$$

Step 3: Calculate the score function $\aleph = [\aleph_{ij}]_{x*y}$ of each q-ROMFSN by using Eq.(1).

$$\begin{bmatrix} 0.1405 & 0.2086 & 0.3346 & 0.4754 & 0.6262 \\ 0.0275 & 0.1985 & 0.2666 & 0.4754 & 0.723 \\ 0.1985 & 0.2666 & 0.3494 & 0.723 & 0.723 \\ 0.0275 & 0.1985 & 0.6262 & 0.723 & 0.806 \end{bmatrix}$$

Step 4: Standardize the score matrix U to obtain its normalized form as $\aleph' = [\aleph'_{ij}]_{x*y}$ by Eq.(2). The criteria h_1, h_2, h_3 are beneficial-oriented and h_4, h_5 are non-beneficial-oriented.

$$\begin{bmatrix} 0.6614 & 0.1483 & 0.1910 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0.4616 \\ 1 & 1 & 0.2303 & 0 & 0.4616 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Step 5: Calculate the standard deviations by Eq. (3), shown in Table 4.

Table 4. Standard deviations

σ_1	σ_2	σ_3	σ_4	σ_5
0.4323	0.4160	0.3822	0.5	0.3541

Step 6: Calculate the correlation between criteria by using Eq.(4).

$$\delta_{jl} = \begin{pmatrix} 1 & 0.95 & 0.80 & 0.79 & 0.57 \\ 0.95 & 1 & 0.69 & 0.69 & 0.38 \\ 0.80 & 0.69 & 1 & 0.53 & 0.74 \\ 0.79 & 0.69 & 0.53 & 1 & 0.66 \\ 0.57 & 0.38 & 0.74 & 0.66 & 1 \end{pmatrix}$$

Step 7: Calculate the informational value of each criterion by Eq.(5), shown in Table 5.

Table 5. The informational value of each criterion

z_1	z_2	z_3	z_4	z_5
0.3847	0.5366	0.4739	0.6650	0.5843

Step 8: Calculate the objective weights of each criterion by Eq.(6), shown in Table 6.

Table 6. Objective weights

ϖ_1	ϖ_2	ϖ_3	ϖ_4	ϖ_5
0.145	0.203	0.179	0.252	0.221

Step 9: Let us consider the subjective weight, as directly presented by decision-makers, is $w = \{0.10,0.15,0.20,0.25,0.30\}$, satisfying $\sum_{j=1}^{y} w_j = 1, 0 \leq w \leq 1$. The objective weight, computed using Eq. (6), is $\varpi = \{0.10,0.15,0.20,0.25,0.30\}$, satisfying $\sum_{j=1}^{y} \varpi_j = 1, 0 \leq \varpi \leq 1$. Calculate the integrated weight ξ by Eq.(7), shown in Table 7.

Table 7. Integrated weights

ξ_1	ξ_2	ξ_3	ξ_4	ξ_5
0.0669	0.1391	0.1645	0.3251	0.3035

Step 6: Calculate the sum of the weighted comparability sequence as A_i by Eq.(8), shown in Table 8.

Table 8. Comparability Sequence A_i

A_1	A_2	A_3	A_4
0.72887	0.4648	0.3835	0.1645

Step 7: Calculate the sum of the power weight of the comparability sequence as B_i by Eq.(9), shown in Table 9.

Table 9. Comparability Sequence B_i

B_1	B_2	B_3	B_4
4.519	1.7903	1.575	1

Step 8: The three aggregation strategies formulated using Eqs.(10)–(12), are presented in Table 10.

Table 10. Appraisal score values

γ_{1a}	γ_{2a}	γ_{3a}	γ_{4a}
0.72887	0.4648	0.3835	0.1645
γ_{1b}	γ_{2b}	γ_{3b}	γ_{4b}
4.8827	4.616	3.906	2
γ_{1c}	γ_{2c}	γ_{3c}	γ_{4c}
1	0.4297	0.3732	0.2219

Step 9: Obtain the evaluation value γ_i by Eq.(13), presented in Table 11.

Table 11. Final values for ranking

γ_1	γ_2	γ_3	γ_4
3.4665	2.5020	2.1330	1.1421

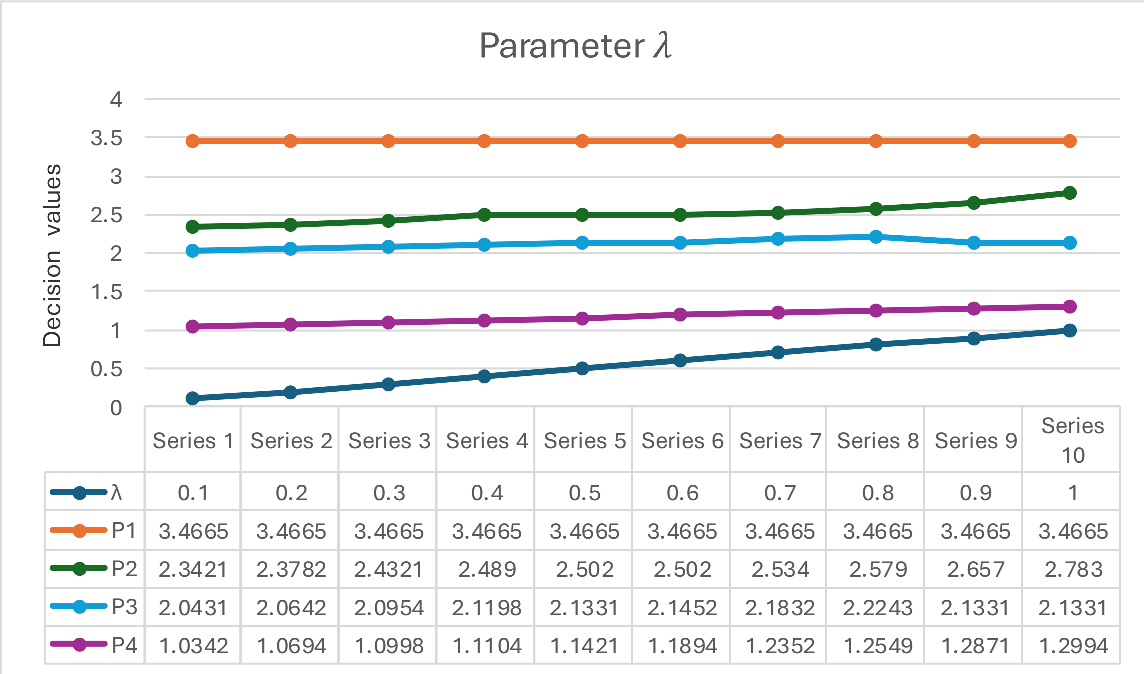
Step 10: Order the alternatives in descending order based on the assessment value γ_i as

$$P_1 > P_2 > P_3 > P_4$$

Thus, it can be concluded that P_1 represents the optimal energy management policy.

5. SENSITIVITY ANALYSIS

Sensitivity analysis is a crucial technique in optimization and decision-making, allowing researchers to assess how variations in input parameters affect model outcomes. In this study, we examine the impact of parameter λ, derived from the CoCoSo approach, on the final ranking of energy management policies. Our findings indicate that λ directly influences the decision values, particularly for P_1, which remains consistent at 3.4665 due to its dependency on Eq.(12). This is because A_1 and B_1 represent the maximum values of A_i and B_i in policy P_1, ensuring that Eq.(12) evaluates to 1. Furthermore, as λ increases, the decision values for P_2, P_3, and P_4 also rise. This trend can be attributed to the role of γ_{1c} in Eq.(12) and the relationship between B_i and A_i in Example 4.1. The variations in these values illustrate the dynamic impact of λ on different policies, necessitating a deeper discussion of its implications, which follows in the subsequent section. Fig. 4 shows the sensitivity of parameter λ.



	Series 1	Series 2	Series 3	Series 4	Series 5	Series 6	Series 7	Series 8	Series 9	Series 10
λ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
P1	3.4665	3.4665	3.4665	3.4665	3.4665	3.4665	3.4665	3.4665	3.4665	3.4665
P2	2.3421	2.3782	2.4321	2.489	2.502	2.502	2.534	2.579	2.657	2.783
P3	2.0431	2.0642	2.0954	2.1198	2.1331	2.1452	2.1832	2.2243	2.1331	2.1331
P4	1.0342	1.0694	1.0998	1.1104	1.1421	1.1894	1.2352	1.2549	1.2871	1.2994

Fig. 4. Sensitivity analysis

5.1 Comparative Analysis

In evaluating the Lq* q-ROMFSS CRITIC-CoCoSo technique against other fuzzy soft set-based decision-making methods such as IFSS [27], PFSS [28], HFSS [29], and Picture FSS [30]. Table 11 presents the comparative analysis of proposed methodology.

- Compared to traditional fuzzy MCDM methods, Lq* q-ROMFSS CRITIC-CoCoSo offers a more structured approach to identifying critical factors and analyzing their multi-dimensional interdependencies in complex systems.
- The IFSS CRITIC-CoCoSo method is constrained by the requirement that the sum of membership and non-membership degrees cannot exceed 1. In contrast, Lq* q-ROMFSS CRITIC-CoCoSo eliminates this restriction, making it better suited for large-scale, complex decision-making scenarios.
- The PFSS CRITIC-CoCoSo technique restricts the square sum of membership and non-membership degrees to a maximum of 1, limiting its ability to handle uncertainties. Lq* q-ROMFSS CRITIC-CoCoSo, however, integrates subjective preferences and uncertainties more effectively, leading to more robust decision-making.
- Techniques like HFSS, Interval-Valued q-ROFSs, Picture FSSs CRITIC- CoCoSo struggle with incomplete and uncertain data. The proposed CRITIC-CoCoSo method addresses this challenge by accommodating both complete and incomplete information, enhancing decision accuracy.

Table 11. Comparison For Proposed Lq* q-ROMFSS CRITIC-CoCoSo

Methods	Truthiness	Falsehood	Loss of Information	Managing multi-dimensional data
Intuitionistic fuzzy soft CRITIC-CoCoSo [27]	✓	✓	✓	X
Pythagorean fuzzy soft CRITIC-CoCoSo [28]	✓	✓	✓	X
Hesitant fuzzy soft CRITIC-CoCoSo [29]	✓	✓	✓	X
Picture fuzzy soft CRITIC-CoCoSo [30]	✓	✓	✓	X
Proposed Lq* q-ROMFSS CRITIC-CoCoSo	✓	✓	✓	✓

Advantages and Limitations are:

1. We introduce a new score function for q-rung orthopair multi-fuzzy soft numbers that considers multi-memF, non-memF, and hesitancy, improving decision-making precision.
2. The Lq∗ q-ROMFSS-MCDM model was chosen for its ability to handle complex decision-making with multi-dimensional fuzzy information and uncertainty. It integrates multi-membership and multi-non-membership values with lattice ordering parameters, making it highly adaptable to real-world problems.
3. Current Lq* q-ROMFSS MCDM methods often struggle with low discrimination, counterintuitive results, and parameter restrictions, making them less effective in selecting the optimal alternative. The CoCoSo method offers a practical solution for handling uncertain information.
4. Existing weight-determination methods focus either on subjective or objective weights, neglecting the integration of both. This work proposes a combined approach to incorporate expert preferences and assessment data better.

Despite its strengths, the proposed method has some limitations:

1. The Lq∗ q-ROMFSS CRITIC-CoCoSo approach struggles to handle uncertain information where both multi-MemF and multi-NMemF are equal to 1.
2. It is less effective in handling diverse, uncertain environments.

6. CONCLUSION

This study presents a Lq* q-rung orthopair multi-fuzzy soft (Lq* q-ROMFSS) CRITIC-CoCoSo approach for selecting optimal energy management policies in smart grids. The proposed method ensures a balanced alignment between energy supply and demand while adhering to key economic, reliability, and safety constraints. The integration of lattice ordering structures introduces a new dimension to computational intelligence, fuzzy modeling, and decision-making processes. This research develops an advanced score function to enhance the comparison of q-rung orthopair multi-fuzzy soft numbers. A case study is conducted to validate the effectiveness of the CRITIC-CoCoSo method in optimizing energy management policy selection within the *Lq* q-ROMFSS* framework. Furthermore, a detailed comparative and sensitivity analysis is performed, benchmarking the CRITIC-CoCoSo approach against existing methodologies. This thorough evaluation provides insights into the strengths, limitations, and practical implications of the proposed technique, offering valuable guidance for decision-makers in the energy sector.

For future research, we recommend exploring the potential applications of the proposed hybrid models across various domains and developing extensions, particularly those incorporating q-rung orthopair multi-fuzzy soft numbers with lattice ordering criteria. Additionally, involving a broader group of managers and experts in the decision-making process would enhance the representativeness and reliability of the results. Despite these future directions, the effectiveness and practicality of the proposed hybrid model remain undeniable.

CONFLICTS OF INTEREST

The authors declare no conflict of interest.

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